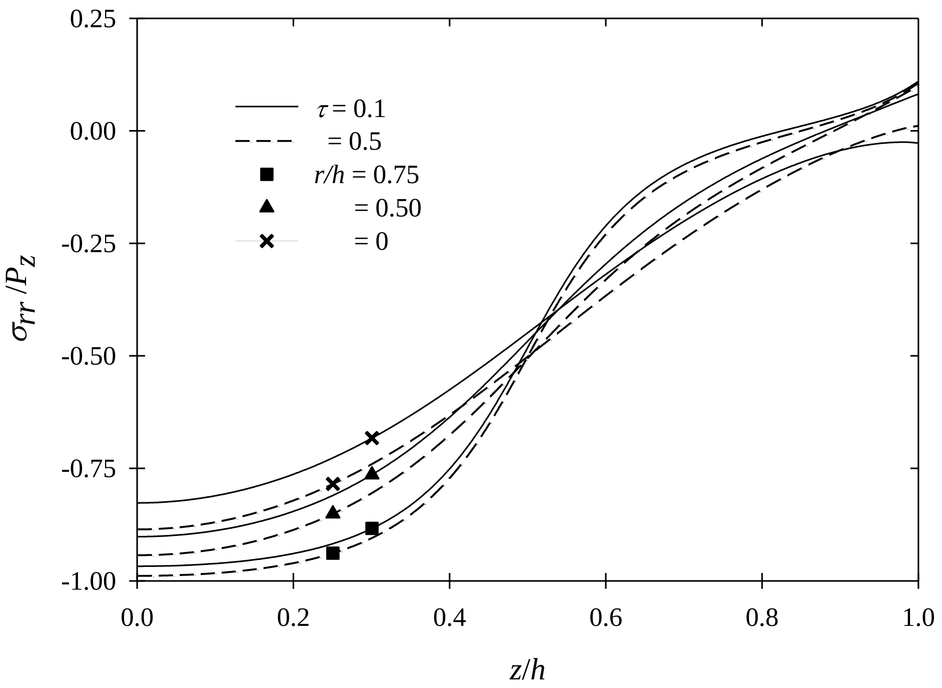

Figure 7. Radial stress along z-direction due to uniform temperature and banded mechanical loading (CdSe, $h_0 = h/2$, $a/h = 1$).

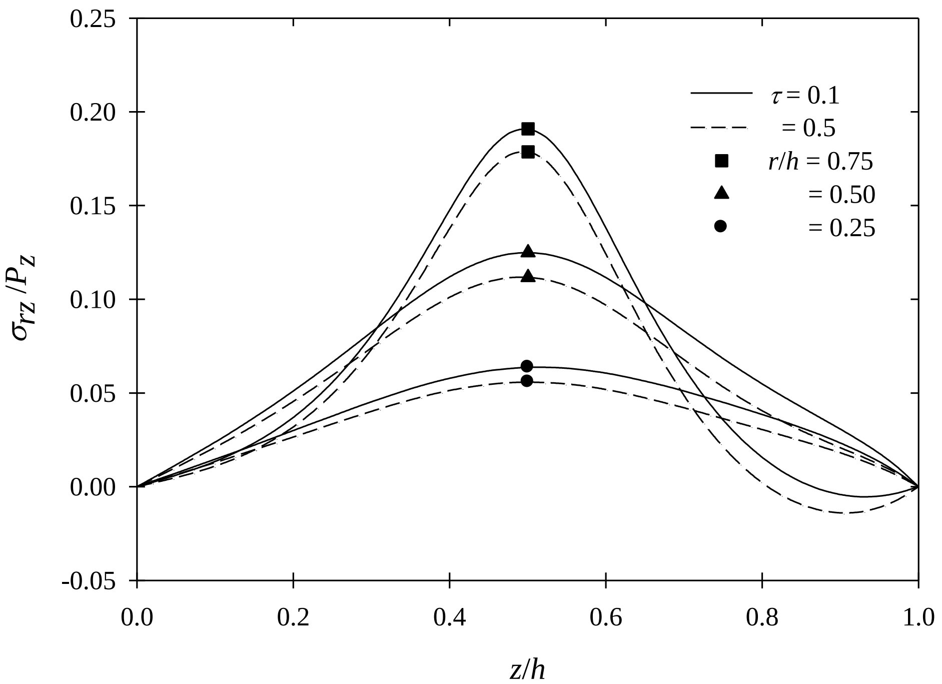

Figure 8. Shear stress along z-direction due to uniform temperature and banded mechanical loading (CdSe, $h_0 = h/2$, $a/h = 1$).

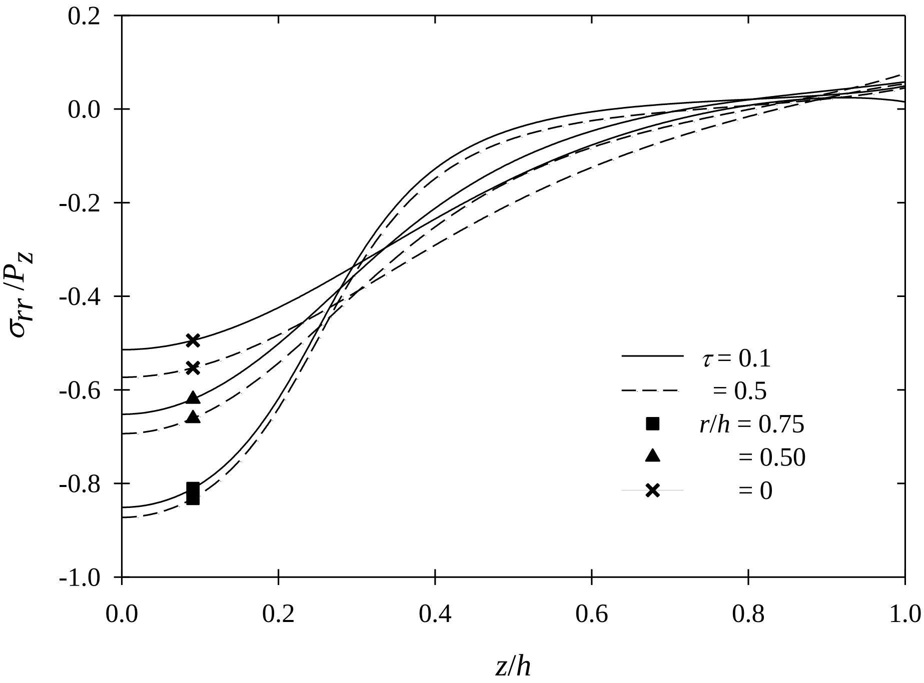

Figure 9. Radial stress along z-direction due to uniform temperature and banded mechanical loading (CdSe, $h_0 = h/4$, $a/h = 1$).

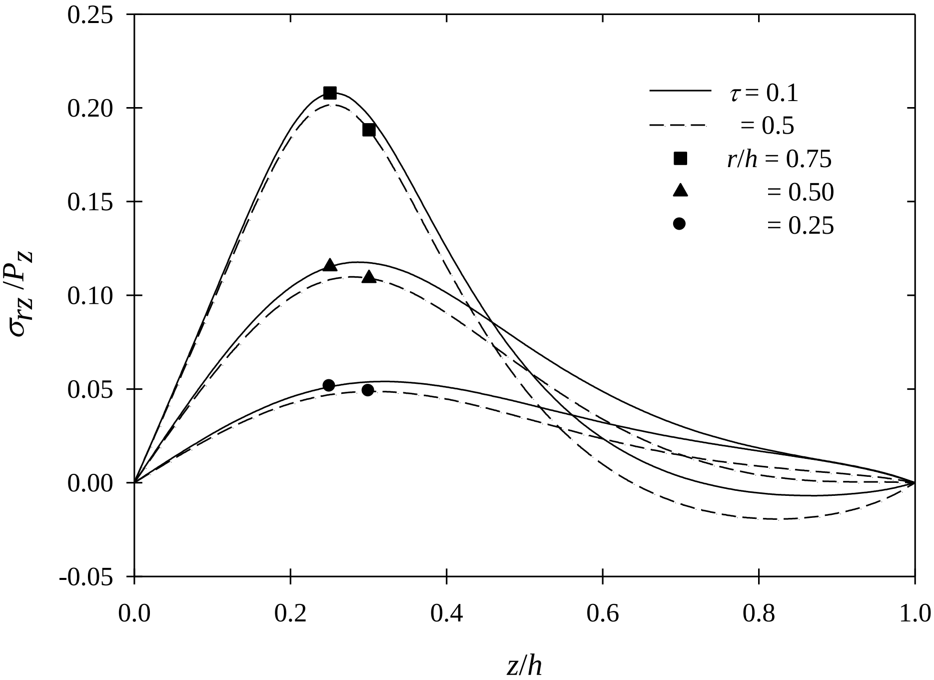

Figure 10. Shear stress along z-direction due to uniform temperature and banded mechanical loading (CdSe, $h_0 = h/4$, $a/h = 1$).

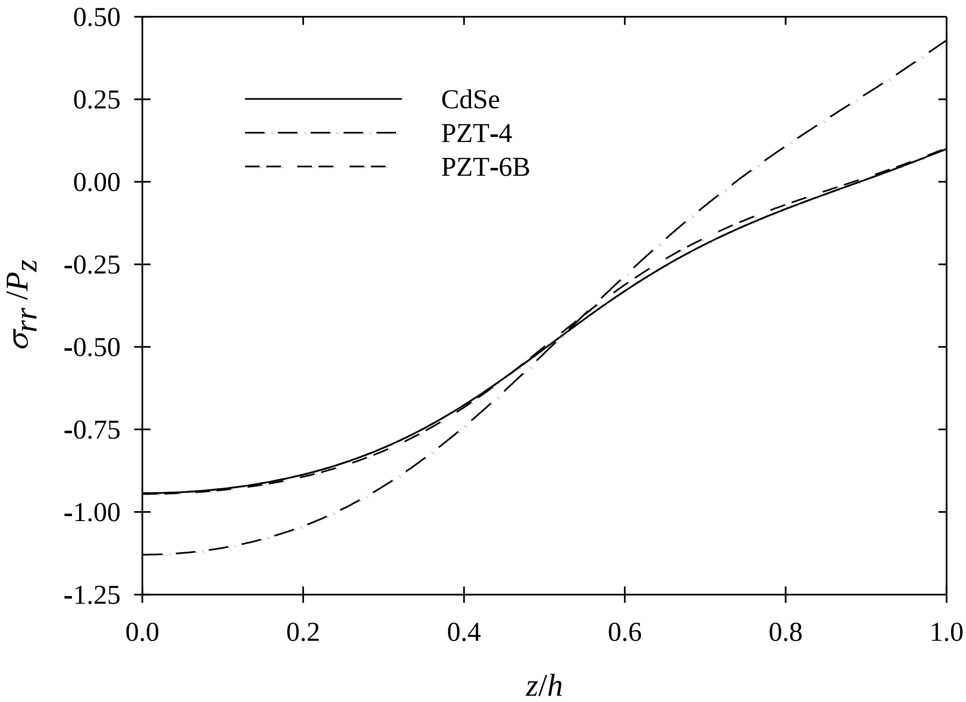

Figure 11. Radial stress along the *z*-direction due to uniform temperature and banded mechanical loading ($h_0 = h/2$, $a/h = 1$).

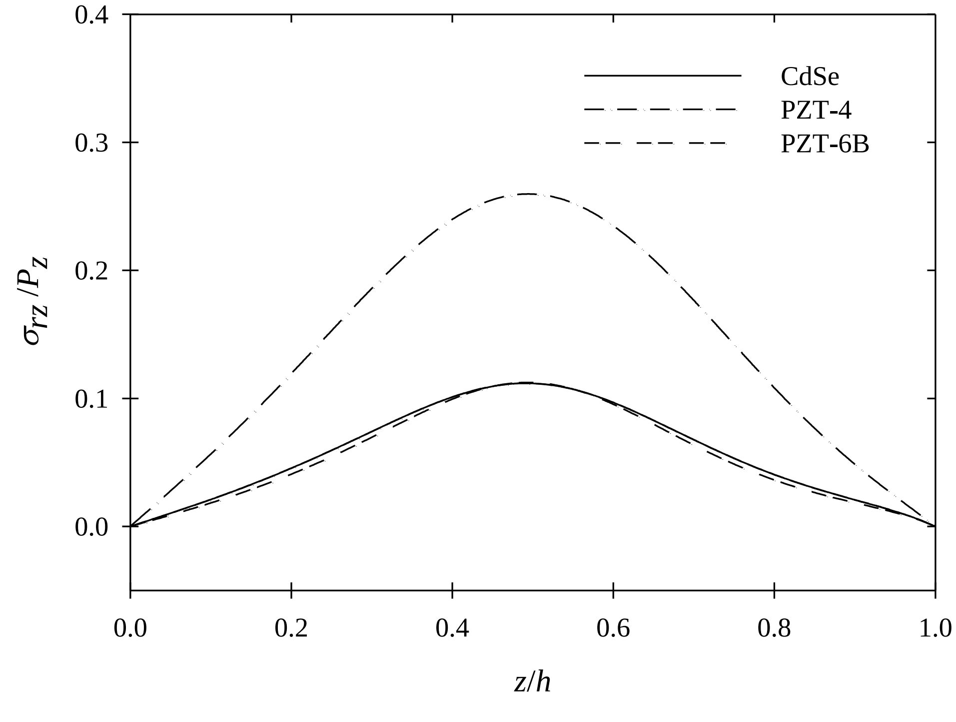

Figure 12. Tangential stress along the *z*-direction due to uniform temperature and banded mechanical loading ($h_0 = h/2$, $a/h = 1$).

Selected numerical solutions for finite solid thermopiezoelectric cylinder as shown in Fig.1 are presented next to portray the salient features of themopiezoelectric responses for a piezoelectric finite cylinder under thermoelectromechanical loading. All numerical results presented hereafter correspond to cases where uniform temperature $\theta(t) = T_1 H(t)$ and radial traction $\sigma_{rr} = P_z H(t)$, where T_1 and P_z are constants, are simultaneously applied over an outer lateral surface of a thermopiezoelectric cylinder with $h = a$. Piezoelectric material properties used in the numerical study are given in Table 2.

Figures 5 and 6 show radial displacements and radial stresses on the $z = 0$ plane in the r-direction respectively for a finite solid cylinder made of Cadmium Selenide (CdSe). It can be seen from Figs. 5 and 6 that the radial displacement and radial stress of the cylinder are varied with time before reaching the steady state condition when $\tau = 2$. Radial displacements due to applied traction and temperature change are almost linear with respect to the radial axis. At the steady state, thermal stress (stress due to non-uniform distribution of temperature) is disappeared and hence stresses in the cylinder are solely due to the applied traction. The deformations of the cylinder at the steady state are, however, influenced by both temperature change and mechanical loading.

Next, the case of a thermopiezoelectric finite cylinder of CdSe subjected to banded radial traction under uniform temperature is considered. The function of banded mechanical loading is given by

$$\bar{P}(z,t) = \begin{cases} -P_z H(t), & |z| \le h_0 \\ 0, & h_0 < |z| \le h \end{cases} \tag{20}$$

Two cases of banded mechanical loading are considered in the numerical study, i.e., $h_0 = h/2$ in Figs. 7 and 8, and $h_0 = h/4$ in Figs. 9 and 10 respectively. Under this loading case, larger number of series for converged solutions is required, i.e. $M \ge 50$. The numerical results are presented for various radial distances, r/h, at different nondimensional times, τ, to show stress distributions along the vertical direction in thermopiezoelectric cylinders at transient state. The numerical results presented in Figs. 7-10 reveal non-uniform distribution of radial and shear stresses along the z-direction. Similar trends of stress distribution are observed from both loading cases at all presented radial distances. The maximum radial and shear stresses are found at $r/h = 0.75$. In addition, the magnitudes of radial stresses for the case $h_0 = h/2$ are evidently higher than those of $h_0 = h/4$, whereas the maximum shear stresses generated in thermopiezoelectric cylinders under $h_0 = h/2$ and $h_0 = h/4$ are not significantly different.

Figures 11 and 12 show radial and tangential stresses respectively for a finite thermopiezoelectric solid cylinder subjected to uniform temperature and banded mechanical loading ($h_0 = h/2$). The numerical results are presented in Figs. 11 and 12 for three different types of piezoelectric materials, namely, CdSe, PZT-4 and PZT-6B (see Table 2) and $\tau = 2$. It can be seen from numerical results in Figs. 11 and 12 that the material properties have a significantly influence on the cylinder responses. In particular, stresses in PZT-4 cylinder are generally higher than those of CdSe and PZT-6B cylinders.

Conclusion

In this paper, a comprehensive analysis of a finite thermopiezoelectric solid cylinder under axisymmetric electromechanical and thermal loading is presented. The formulation of a boundary-value problem is based on general solutions of a thermopiezoelectric cylinder, which are derived by employing a generalized displacement potential function method together with a Fourier-Bessel series expansion. Current numerical solutions agree well with the existing solutions for the limiting case of a thermoelastic cylinder. Numerical results of a finite solid thermopiezoelectric cylinder are shown for different loading conditions and material types. The present general solution, and the analytical procedure outlined in this paper can be further extended to solve more complicated boundary-value problems involving finite hollow cylinders and composite cylinders.

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