Invited Paper

A PROTOTYPE SYSTEM TO FIND THE SERIES RESONANT FREQUENCY OF A QUARTZ CRYSTAL USING IMPEDANCE MEASUREMENT

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Abstract
An implementation technique for measuring the resonant frequency of a quartz crystal is presented. It applies a sinusoidal voltage to the quartz crystal at selected frequencies and measures its current which is used to calculate its impedance. Frequency scanning and interval bisection with a final resolution of 1 Hz are performed until the minimum impedance is reached at the resonant frequency. Two key hardware components are direct digital synthesizer (DDS) for generating a pure sinusoidal signal at a precise frequency and a two channels 12 bit analog to digital converter for converting the voltage and current waveforms into a sequence of digital values. Other key software algorithms include frequency selection to program the DDS and sinusoidal curve fitting to obtain the sine amplitude accurately from the sampled data. An experiment with a commercial 5 MHz quartz crystal is carried out yielding 4,984,145 Hz resonant frequency. One application of this technique is in the use of quartz crystal as a tiny mass sensor from which its resonant frequency varies with the applied mass.

Keywords: Direct Digital Synthesizer, Microbalance, Quartz Crystal, Resonant Frequency

Introduction

A quartz crystal [1] is an oxide of silicon $\text{SiO}_2$ in crystal form. It can be manufactured with precise dimension and orientation and placed between two electrodes as shown in Figure 1.

Figure 1. Quartz crystal in its original form (left), as a commercial product with metal can and electrodes (middle) and its electrical symbol (right)

Due to its piezoelectric property, a quartz crystal vibrates when a voltage is applied across its electrodes. This frequency of vibration is called the resonant frequency [2] and depends on the physical property and dimension of the quartz crystal. With small temperature sensitivity
and low acoustic loss, its resonant frequency is highly stable when compared to other elements such as LC tank [3], [4]. For example a 10MHz quartz crystal may exhibit only 60ppm change in its resonant frequency for a temperature range between -60 to 100 Celcius. Although this property is now being challenged by a new element called MEMs resonator, a quartz crystal is still commonly found in any product that requires precise and stable timing such as digital watch, radio transmitter, microprocessor and etc.

This paper discusses another special application of quartz crystal in measuring small mass by applying it on the surface of the quartz crystal which slightly lowers its resonant frequency according to the following formulae proposed by Sauerbrey [5]

\[ \Delta f = -\frac{2f_s^2 \Delta m}{A\sqrt{\rho \mu}} \]  

where

- \( \Delta m \) = load mass
- \( f_s \) = resonant frequency of the quartz crystal in its fundamental mode
- \( \Delta f \) = change in the resonant frequency due to the load mass
- \( A, \rho, \mu \) = surface area, quartz density, shear modulus of the quartz crystal

For example a 10MHz quartz crystal with 1 cm\(^2\) a surface area, 2.648 g·cm\(^{-3}\) mass density and 2.947x10\(^{11}\) g·cm\(^{-1}\)·s\(^{-2}\) shear modulus will exhibit a resonant frequency change of -226.4Hz when subjected to a 1 microgram load mass. This high sensitivity makes it suitable for measuring very small load that cannot be achieved by using normal scales. Some examples are monitoring deposition rate of thin film [10] and weighing viruses or polymers [11] typically needed in biological and chemical sciences. Such a weighing equipment is called quartz crystal microbalance or QCM which can measure mass as small as 1 microgram per sq.cm. by measuring its change in resonant frequency.

There are 3 methods that had been proposed in the literature to measure the quartz resonance frequency. The oscillation measurement method, shown in Figure 2a, employs the quartz crystal in a gate oscillator [8] and measures its oscillating frequency with a counter. The impedance measurement method, shown in Figure 2b, uses a network analyzer (NWA) [6] to find the frequency at which the measured impedance of the quartz crystal is smallest. The zero phase measurement method, shown in Figure 2c, uses a network analyzer with added LC transformation circuits [7] to detect the frequency at which the admittance of the quartz crystal has a zero phase.

![Diagram](image)

Figure 2. Methods for measuring resonant frequency: Oscillation measurement (left), Impedance measurement (middle) and Zero phase measurement (right)

Among these 3 methods, the oscillation measurement is very popular due to its simplicity and low cost of implementation. However the circuit oscillates at a frequency that can be 1%...
higher than the resonant frequency and hence making the QCM measurement problematic. Being an instrument for general network measurement, the network analyzer used in both the impedance and zero phase measurements are typically very expensive although the zero phase method can also measure the resistant loss of the quartz crystal at the expense of additional LC transformation circuits.

This paper presents the design of a low cost circuit and system for finding the resonant frequency using the impedance measurement method without the need of a network analyzer. It employs new electronic chips like a direct digital synthesizer (DDS), analog to digital converter (ADC) and microcontroller (MCU) to perform the required operation and use a person computer for calculation. To provide a basic concept behind the impedance measurement method, the circuit model of a quartz crystal is revised in Section II and formulae for its series and parallel resonant frequency are given. Then the proposed method is described in Section III followed by the implementation in Section IV. Finally the result and conclusion are given in Section V.

**Quartz Crystal Circuit Model**

Figure 3. shows the symbol and the Butterworth-van-Dyke (BvD) equivalent circuit [9] of a quartz crystal vibrating in the fundamental mode. The top part of the equivalent circuit models the motional properties of a quartz crystal where L is the inductance proportional to the mass, C is the capacitance inversely proportional to the stiffness and R is the resistance due to the dissipative loss. The bottom part consists of $C_0$ which models the capacitance of the dielectric between its electrodes. The values of these circuit elements depend on the elastic constant, piezoelectric constant, crystal cuts, type of mechanical wave, dielectric constant, surface area and thickness of the quartz crystal.

This circuit model lends itself to 3 important parameters of the quartz crystal. They are series resonant frequency $f_s$, parallel resonant frequency $f_p$ and quality factor $Q$ given by

$$f_s = \frac{1}{2\pi\sqrt{LC}} ;$$
$$f_p = \frac{1}{2\pi\sqrt{LC}} \left( 1 + \frac{C}{C_0} \right)^{1/2} ;$$
$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

The following table lists the values of the parameter for a few commercial quartz crystals.
Table I. Parameters of some Commercialized Quartz Crystals [12]

<table>
<thead>
<tr>
<th>$f_0$ (kHz)</th>
<th>L (mH)</th>
<th>C (pF)</th>
<th>R (Ω)</th>
<th>$C_0$ (pF)</th>
<th>$f_s$ (kHz)</th>
<th>$f_p$ (kHz)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.000</td>
<td>0.005</td>
<td>40.000</td>
<td>7</td>
<td>20</td>
<td>20.007</td>
<td>12.500</td>
</tr>
<tr>
<td>200</td>
<td>27</td>
<td>0.024</td>
<td>2.000</td>
<td>10</td>
<td>200</td>
<td>200.240</td>
<td>17.000</td>
</tr>
<tr>
<td>450</td>
<td>8.5</td>
<td>0.015</td>
<td>1.050</td>
<td>5</td>
<td>450</td>
<td>450.675</td>
<td>23.200</td>
</tr>
<tr>
<td>1 MHz</td>
<td>3.16</td>
<td>0.008</td>
<td>400</td>
<td>3.2</td>
<td>1 MHz</td>
<td>1.001MHz</td>
<td>50.000</td>
</tr>
<tr>
<td>2 MHz</td>
<td>0.528</td>
<td>0.012</td>
<td>100</td>
<td>4</td>
<td>2 MHz</td>
<td>2.030MHz</td>
<td>66.000</td>
</tr>
<tr>
<td>4 MHz</td>
<td>0.21</td>
<td>0.007</td>
<td>22</td>
<td>2.4</td>
<td>4 MHz</td>
<td>4.006MHz</td>
<td>240.990</td>
</tr>
<tr>
<td>5 MHz</td>
<td>0.03</td>
<td>0.033</td>
<td>10</td>
<td>5</td>
<td>5 MHz</td>
<td>5.016MHz</td>
<td>94.200</td>
</tr>
<tr>
<td>8 MHz</td>
<td>0.014</td>
<td>0.027</td>
<td>8</td>
<td>5.6</td>
<td>8 MHz</td>
<td>8.019MHz</td>
<td>88.680</td>
</tr>
<tr>
<td>10 MHz</td>
<td>0.0101</td>
<td>0.025</td>
<td>5</td>
<td>5.5</td>
<td>10 MHz</td>
<td>10.002MHz</td>
<td>126.000</td>
</tr>
<tr>
<td>15 MHz</td>
<td>0.00417</td>
<td>0.028</td>
<td>5</td>
<td>4</td>
<td>15 MHz</td>
<td>15.052MHz</td>
<td>78.500</td>
</tr>
<tr>
<td>30 MHz</td>
<td>0.0101</td>
<td>0.0027</td>
<td>45</td>
<td>6</td>
<td>30 MHz</td>
<td>30.007MHz</td>
<td>42.300</td>
</tr>
<tr>
<td>150 MHz</td>
<td>0.00281</td>
<td>0.0004</td>
<td>70</td>
<td>6</td>
<td>150 MHz</td>
<td>150.005MHz</td>
<td>37.887</td>
</tr>
</tbody>
</table>

Note that $f_s$ is also the vibration frequency of the quartz crystal. Since the load affects only the motional properties, it is $f_s$ that must be measured by a QCM. The table shows also that Q is several orders of magnitude higher than the same circuit built from discrete LC components.

The minimum impedance measurement method

From the circuit model in Figure 3, the total impedance $Z$ of a quartz crystal as a function of frequency $f$ is given by

$$Z = \left( R + \frac{1}{j\omega C} + \frac{1}{j\omega L} \right) \frac{1}{j\omega C_0}$$

where $\omega = 2\pi f$. Figure 3. shows the impedance (in log scale) versus frequency plot of a 10MHz quartz crystal using the parameter values in Table I.

![Figure 4. Impedance of a 10MHz quartz crystal versus frequency](image-url)
The plot shows two frequencies at which the impedance reaches minimum and maximum peaks. Since the quality factor of a quartz crystal is extremely high, it can be shown that the minimum impedance occurs at \( f_s \) and the maximum impedance occurs at \( f_p \). This is the basic concept behind the impedance measurement method which is shown in Figure 5. In this method, a sinusoidal voltage of a given frequency is applied to a quartz crystal and its current is measured. The impedance is calculated by dividing the voltage amplitude by the current amplitude. The frequency is then scanned until the calculated impedance reaches the minimum.

\[
\text{Vsin}(2\pi ft) \quad \text{Quartz Crystal}
\]

\[
\text{Z} \rightarrow
\]

Figure 5. Concept of the impedance measurement

As the quartz crystal exhibits changes in its resonance frequency \([ \_] \) when subjected to mass load. It is then natural to assume that the loaded mass increases the total mass of the quartz crystal. This effect can be included in the BvD model of Figure 3 by adding a small inductance \( \Delta L \) in series with \( L \) \([ \_] \) as shown in Figure 6.

\[
\text{Load mass } \Delta m
\]

\[
\text{R} \quad \text{C} \quad \text{L} \quad \text{Load mass } \Delta m
\]

\[
\text{C}_0
\]

\[
\text{Z}_T
\]

Figure 6. Equivalent circuit model of loaded quartz crystal

Thus the resonant frequency of the loaded quartz crystal becomes

\[
f_s' = \frac{1}{2\pi \sqrt{L'C}} \quad \text{where} \quad L' = L + \Delta L
\]

from which the change in the resonance frequency can be approximated by

\[
\Delta f_s = f_s' - f_s \approx -\frac{\pi f_s^2 \Delta L}{\sqrt{L'C}}
\]

which is similar to equation (1). Figure 7. shows the impedance plot of a 10MHz quartz crystal of Table I at unloaded \( (\Delta L = 0) \) and loaded \( (\Delta L = 5\mu \text{H}) \) conditions. From the graph the
resonant frequency shifts from 10.015887MHz to 10.013409MHz and hence $\Delta f_s$ is 2478Hz. From (2), the calculated value of $\Delta f_s$ is 2478Hz.

![Impedance plot of a 10MHz quartz crystal at unloaded ($\Delta L = 0$) and loaded ($\Delta L = 5\mu H$) conditions](image)

**A prototype circuit for measuring impedance**

Although the impedance measurement method is not new, its implementation can vary significantly both in terms of hardware and software. Rather than using an expensive network analyzer (NWA), our prototype implementation is constructed from advance but low cost electronic chips. The hardware of the prototype is shown in Figure 8. The function of each block in the figure is briefly described as follows:

- **a) Signal Generator.** It consists of AD9385 chip [13] that generates a sinusoidal waveform $V_o$ whose frequency $f$ can be digitally programmed via its serial data input. This waveform drives the quartz crystal under test.
- **b) Microcontroller board MCU1.** It employs a PIC16F887 [14] microcontroller chip that generates the serial data associated with the frequency $f$ to the signal generator.

![Hardware block diagram of the prototype impedance measurement system](image)
c) Current sense resistor $R_s$. It is connected in series with the quartz crystal to sense its current in the form of voltage $V_I$. Its value is small within the same range as the internal resistance of the quartz crystal.


e) Personal computer. It communicates with both MCU boards to set the frequency $f$ and read the converted data. It also performs algorithms to calculate the impedance and adjust the frequency.

Figure 9. shows the photo of the signal generator module based on AD9385 chip along with its schematic diagram. It uses a Direct Digital Synthesizing (DDS) [14] technique and a 12 bit cosine ROM to generate a sinusoidal waveform at frequency $f$ assigned through its serial data input SDATA. The chip operates at 50MHz master clock and accepts a 32 bit frequency data $F$ allowing a frequency resolution of

$$f_r = \frac{50 \times 10^6}{2^{32}} = 0.01164 \text{ Hz/bit}$$

To obtain a 5MHz signal, the $F$ value should be

$$F = \frac{5 \times 10^6}{f_r} = 429496729.6 \approx 429496730$$

![Functional block diagram of AD9385 DDS module](image)

Figure 9. The AD9385 DDS module (left) and its schematic diagram (right).

The resistor $R_s$ senses the current through the quartz crystal in the form of $V_I$ which is used to calculate the impedance $Z_T$ (see Figure 5.) according to

$$Z_T = \frac{V_o}{I} = R_s \frac{V_o}{V_I}$$
Since the Q-factor is extremely high and $\Delta L/L$ is very small, by choosing $R_s$ close to the resistance $R$ of the quartz crystal, it can be shown that the minimum values of $Z_T$ and $Z_s$ (see Figure 4.) occur at almost the same frequency $f_s$.

The analog to digital converter (ADC) within STM32F407VG is programmed to sample $V_o$ and $V_l$ simultaneously and convert them into 12 bit data. However the maximum sampling frequency that can be set is only 1MHz. This means that both $V_o$ and $V_l$ signals whose frequency is around 5MHz will be under-sampled as shown in Figure 10. To obtain the magnitude of their signals, a sine curve fitting is performed on the digitized signal by the computer to yield the desired magnitude.

![Figure 10. A 5MHz signal and its 1MHz under-sampled signal](image)

The search to find the minimum impedance is carried out by the computer. It starts with choosing $F$ well below the resonant frequency $f_s$. Then the frequency value is constantly increased by a small interval $\Delta F$ and the impedance is measured accordingly. From Figure 4., while the frequency is below $f_s$, the impedance will decrease with $F$. This process goes on until the impedance starts to increase indicating that its minimum has been passed. Thus $\Delta F$ must be chosen smaller than $f_p - f_s$ to guarantee that the scanned frequency will fall within $f_p$ and $f_s$ where the impedance is monotonically increasing. Once the interval containing $f_s$ is found, a bisection algorithm is performed to reach the minimum impedance. Since the frequency value is programmed in 32 bit data, at most 32 trials are needed to find the resonant frequency.

**Experiment Result and Conclusion**

An experiment according to the system described in Section IV is set up to measure the resonant frequency of a commercial 5MHz quartz crystal. According to Table I, we have
\( f_p - f_s = 16 \text{ kHz} \) and \( R = 10 \text{ ohm} \). Hence the value of the current sensing resistor \( R_s \) is chosen to be 5ohms, to maintain high Q-factor. The operation frequency starts at 4.9MHz and is scanned with an increment of 1 kHz until the interval containing the minimum impedance is found. This interval is then bisected until the resonant frequency is found at 4,984,145 Hz when the impedance reaches minimum at 25.3333 \( \Omega \).

In conclusion, the minimum impedance method is used to find the series resonant frequency of a quartz crystal. The main contribution of this paper lies in its detailed hardware and algorithm implementation which is low cost. Using this experiment, a QCM for measuring small mass can be constructed. Further hardware improvement includes the addition of a circuit to detect to the peak voltage as oppose to software detection. Finally it should be mentioned that this method as well as the oscillation method may give more measurement errors when the series resistance of the quartz crystal is high, e.g. higher than 1Kohm, because its Q-factor degrades. This situation arises when the mass load has high viscosity. In that situation the zero phase method has proved to solve this problem. It is however our intention to improve the proposed system to handle such case as well in the future.

References
